FILTRE DE MICROUNDE

Proiectarea filtrelor prin metoda pierderilor de inserție

$$P_{LR} = \frac{Puterea\ disponibila\ de\ la\ sursa}{Puterea\ livrata\ sarcinii} = \frac{P_{inc}}{P_L} = \frac{1}{1-\left|\Gamma(\omega)\right|^2}$$

$$IL = 10\log P_{LR}$$

$$\left|\Gamma(\omega)\right|^2 = \frac{M\left(\omega^2\right)}{M\left(\omega^2\right)+N\left(\omega^2\right)}$$

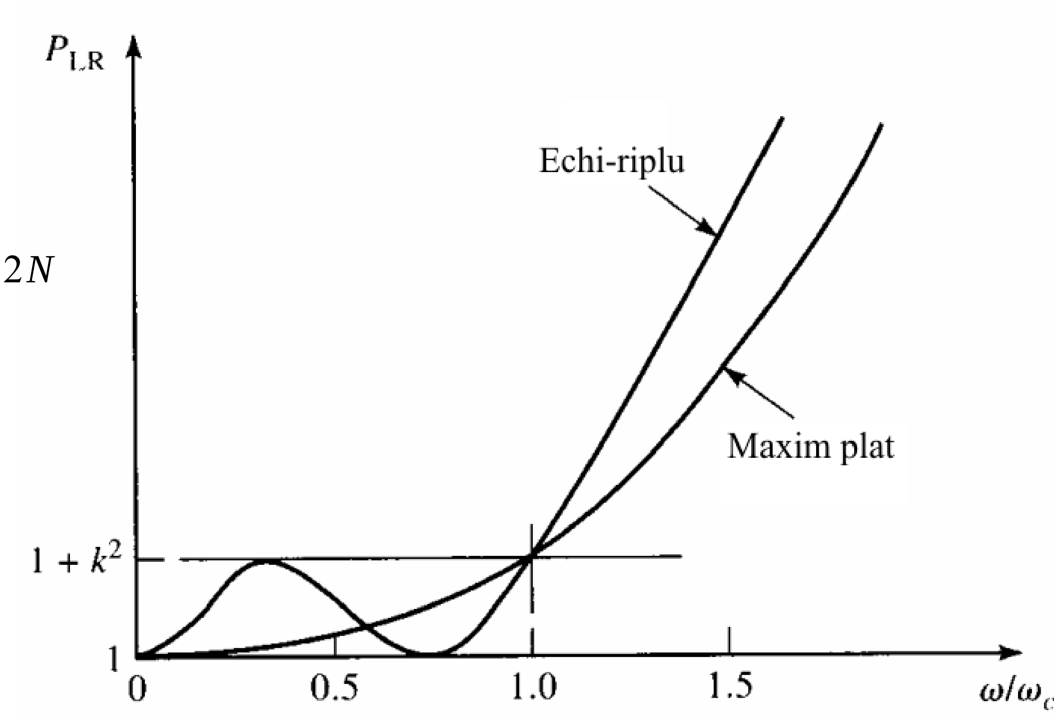
$$P_{LR} = 1+\frac{M\left(\omega^2\right)}{N\left(\omega^2\right)}$$

Tipuri de raspunsuri

Maxim Plat

$$P_{LR} = 1 + k^2 \left(\frac{\omega}{\omega_c} \right)^{2N}$$

$$P_{LR} \approx k^2 (\omega/\omega_c)^{2N}$$

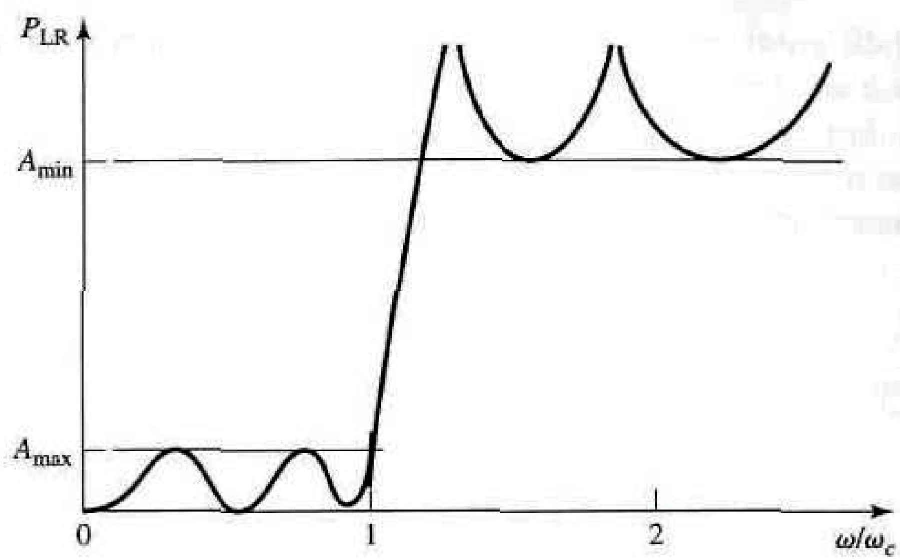


Echi-Riplu

$$P_{LR} = 1 + k^2 T_N^2 \left(\frac{\omega}{\omega_c} \right)$$

$$P_{LR} \approx \frac{k^2}{4} \left(\frac{2\omega}{\omega_c} \right)^{2N}$$

Tipuri de raspunsuri



Eliptic

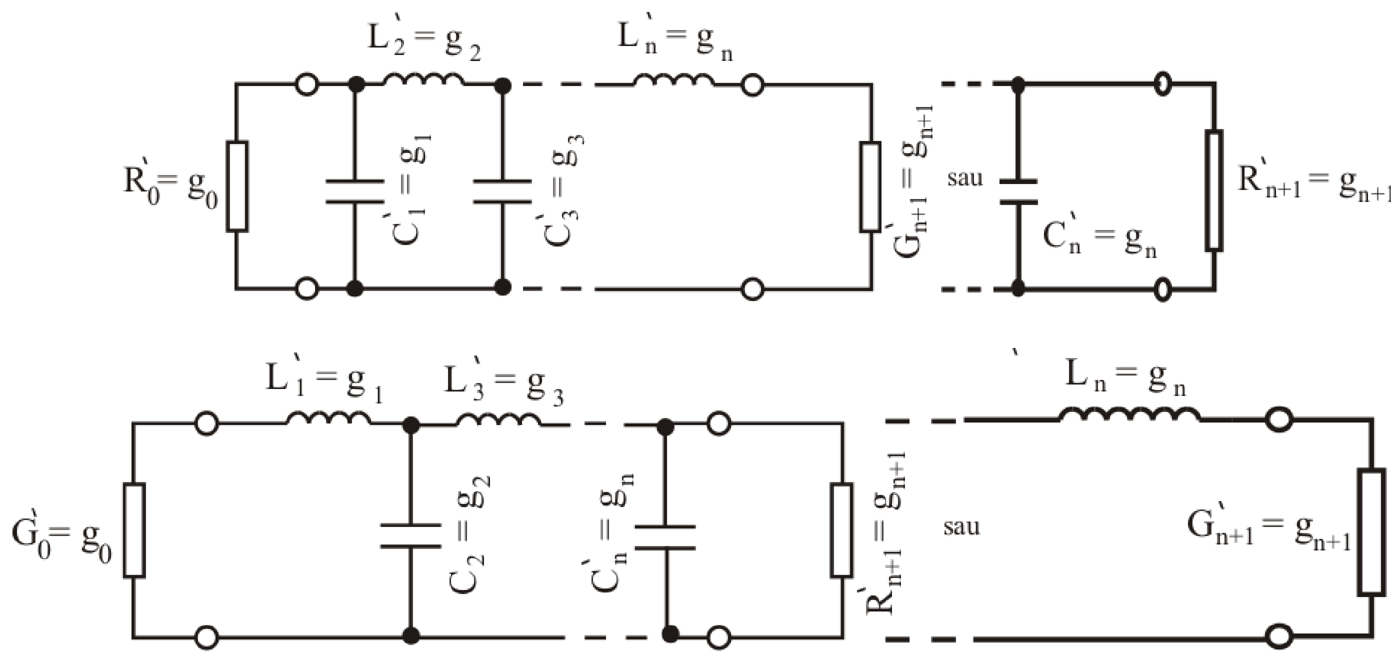
Filtru cu raspuns liniar in faza

$$\phi(\omega) = A\omega\left[1 + p\left(\frac{\omega}{\omega_c}\right)^{2N}\right]$$

$$\tau_d = \frac{d\phi}{d\omega} = A\left[1 + p(2N+1)\left(\frac{\omega}{\omega_c}\right)^{2N}\right]$$

intirzierea de grup

Filtre prototip

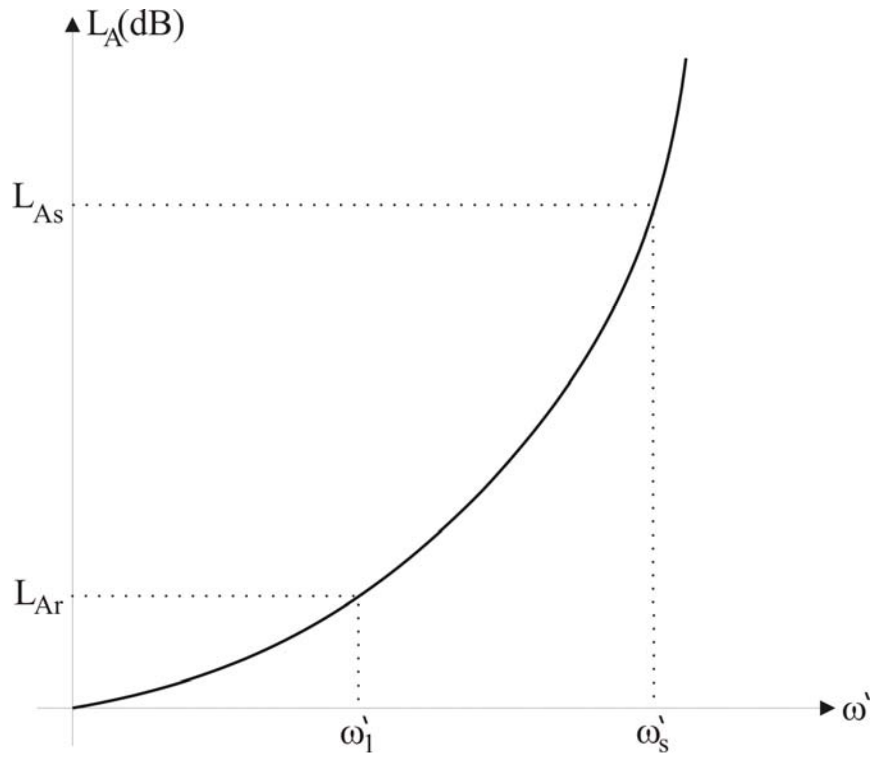


$$g_0 = \begin{cases} \text{rezistenta generatorului } R'_0 \text{ daca } g_1 = C'_1 \\ \text{conductanta generatorului } G'_0 \text{ daca } g_1 = L'_1 \end{cases}$$

$$g_{n+1} = \begin{cases} \text{rezistenta de sarcina } R'_{n+1} \text{ daca } g_n = C'_n \\ \text{conductanta de sarcina } G'_{n+1} \text{ daca } g_n = L'_n \end{cases}$$

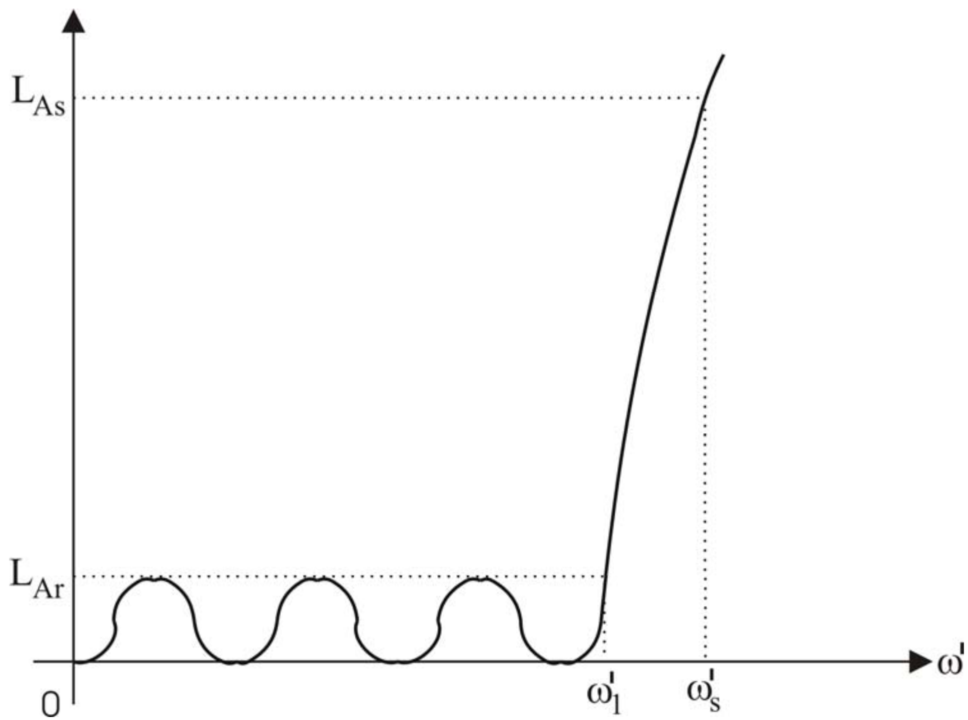
$$g_k\Big|_{k=\overline{1,n}} = \begin{cases} \text{inductanta unei bobine} \\ \text{capacitatea unui condensator paralel} \end{cases}$$

Calculul ordinului filtrului maxim plat



$$n \geq \frac{\log_{10}\left\{\left[10^{\frac{L_{As}}{10}}-1\right]\Big/\left[10^{\frac{L_{Ar}}{10}}-1\right]\right\}}{2\log_{10}\left[\omega'_s/\omega'_1\right]}$$

Calculul ordinului filtrului echi-riplu



$$n \geq \frac{ch^{-1}\left(\sqrt{\left\{\left[\left(10^{0,1L_{As}}\right)-1\right]/\left[\left(10^{0,1L_{Ar}}\right)-1\right]\right\}}\right)}{ch^{-1}\left(\omega'_s / \omega'_1\right)}$$

Filtre prototip de tip maxim-plat cu terminații rezistive

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

$$g_0 = 1$$

$$g_k = 2\sin\left[\frac{(2k-1)\pi}{2n}\right], k = 1,2,3,\ldots, n$$

$$g_{n+1} = 1$$

Filtre prototip de tip echi-riplu cu terminații rezistive

$$g_1 = \frac{2a_1}{\gamma}$$

$$g_k = \frac{4a_{k-1}a_k}{b_{k-1}g_{k-1}}, k = 2,3,\ldots,n$$

$$g_{n+1} = \begin{cases} 1 \quad pentru \ n = impar \\ \coth^2\left(\frac{\beta}{4}\right) \quad pentru \ n = par \end{cases}$$

$$a_k = \sin\left[\frac{(2k-1)\pi}{2n}\right], k = 1,2,\ldots,n$$

$$b_k = \gamma^2 + \sin^2\left(\frac{k\pi}{n}\right), k = 1,2,\ldots,n$$

$$\beta = \ln\left(\coth\frac{L_{Ar}}{17{,}37}\right)$$

$$\gamma = \sinh\left(\frac{\beta}{2n}\right)$$

Filtre prototip de tip echi-riplu cu terminații rezistive

0.5 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

Scalarea in impedanta si frecventa FTJ

Scalarea in impedanta

$$L' = R_0 L$$

$$C' = \frac{C}{R_0}$$

$$R'_s = R_0$$

$$R'_L = R_0 R_L$$

Scalarea in frecventa

$$\omega \leftarrow \frac{\omega}{\omega_c}$$

Valorile noi

$$L'_k = \frac{R_0 L_k}{\omega_c}$$

$$C'_k = \frac{C_k}{R_0 \omega_c}$$

Scalarea in impedanta si frecventa FTS

Scalarea in impedanta

$$L' = R_0 L$$

$$C' = \frac{C}{R_0}$$

$$R'_s = R_0$$

$$R'_L = R_0 R_L$$

Scalarea in frecventa

$$\omega \leftarrow -\frac{\omega_c}{\omega}$$

Valorile noi

$$C'_k = \frac{1}{R_0 \omega_c L_k}$$

$$L'_k = \frac{R_0}{\omega_c C_k}$$

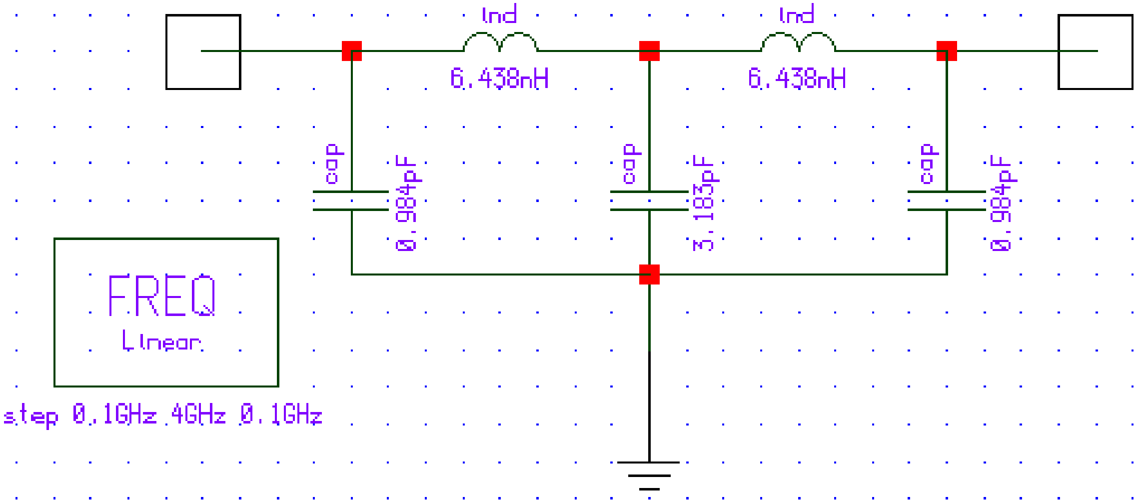
Exemplu

Să se proiecteze un filtru trece-jos de tip maxim plat cu o frecvență de tăiere de 2 GHz, care să lucreze pe 50 Ω, şi să aibă pierderi de inserție de cel puțin 15 dB la 3 GHz. Calculați răspunsul în amplitudine şi în fază între 0 şi 4 GHz. şi comparați-l cu filtrul echi-riplu, cu riplu de 3 db şi de acelaşi ordin.

Solutie

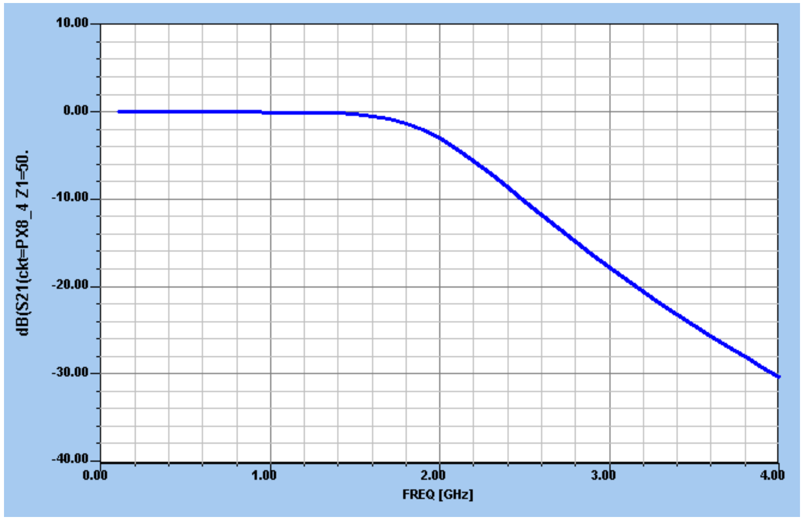
$$N \geq \frac{\log\left(10^{15/10} - 1\right)}{2\log\left(3/2\right)} = 4.22$$

g1 = 0.618
g2 = 1.618
g3 = 2.000
g4 = 1.618
g5 = 0.618

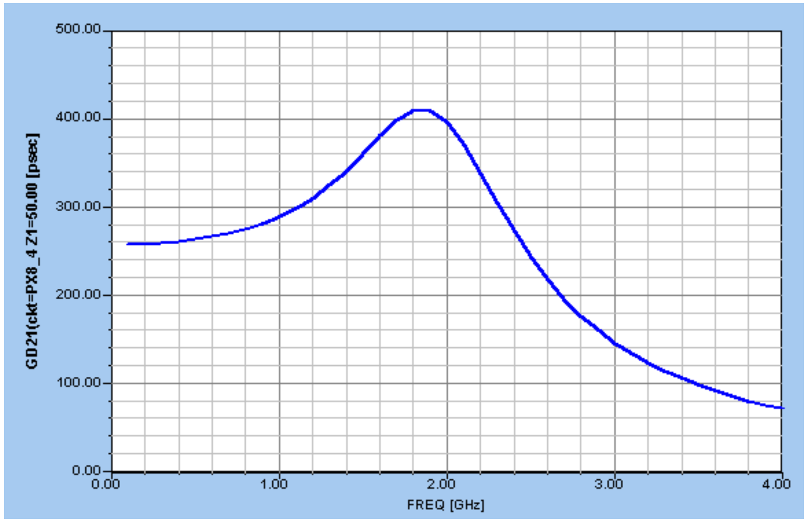


C1' = 0.984 pF,
L2' = 6.438 nH,
C3' = 3.183 pF,
L4' = 6.438 nH,
C5' = 0.984 pF.

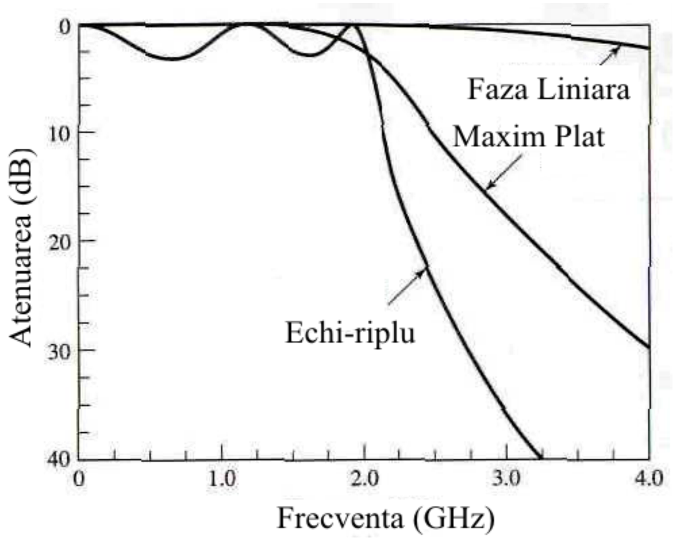
Simularea - 1



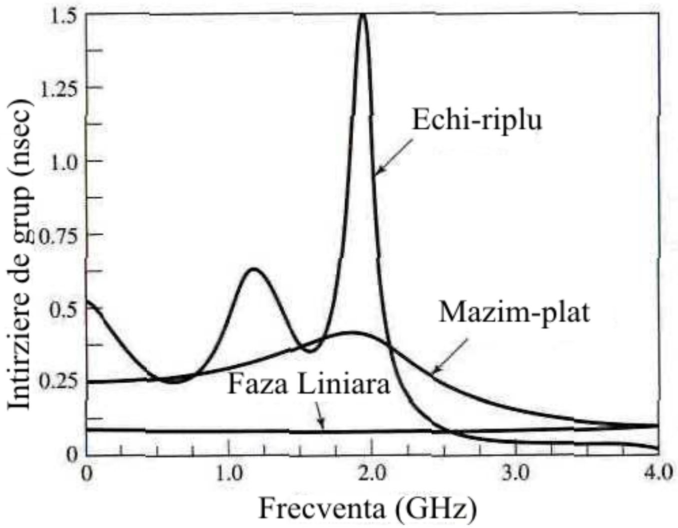
10.00
0.00
-10.00
-20.00
-30.00
-40.00
dB(S21(ckt=PX8_4 Z1=50.
0.00
1.00
2.00
3.00
4.00
FREQ [GHz]



500.00
400.00
300.00
200.00
100.00
0.00
GD21(ckt=PX8_4 Z1=50.00 [psec]
0.00
1.00
2.00
3.00
4.00
FREQ [GHz]



Faza Liniara
Maxim Plat
Echi-riplu
Atenuarea (dB)
0
10
20
30
40
0
1.0
2.0
3.0
4.0
Frecventa (GHz)



Echi-riplu
Mazim-plat
Faza Liniara
Intirziere de grup (nsec)
1.5
1.25
1.0
0.75
0.5
0.25
0
0
1.0
2.0
3.0
4.0
Frecventa (GHz)

Scalarea in impedanta si frecventa
FTJ - FTB

Scalarea in impedanta

$$L' = R_0 L$$

$$C' = \frac{C}{R_0}$$

$$R'_s = R_0$$

$$R'_L = R_0 R_L$$

Scalarea in frecventa

$$\omega \leftarrow \frac{\omega_0}{\omega_2 - \omega_1}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

Valorile noi

Ramura serie

(in serie)

$$L'_k = \frac{L_k}{\Delta \omega_0}$$

$$C'_k = \frac{\Delta}{\omega_0 L_k}$$

Ramura paralel

(in paralel)

$$L'_k = \frac{\Delta}{C_k \omega_0}$$

$$C'_k = \frac{C_k}{\omega_0 \Delta}$$

Scalarea in impedanta si frecventa FTJ - FOB

Scalarea in impedanta

$$L' = R_0 L$$

$$C' = \frac{C}{R_0}$$

$$R'_s = R_0$$

$$R'_L = R_0 R_L$$

Scalarea in frecventa

$$\omega \leftarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^{-1}$$

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

Valorile noi

Ramura serie

(in paralel)

$$L'_k = \frac{\Delta L_k}{\omega_0}$$

$$C'_k = \frac{1}{\omega_0 \Delta L_k}$$

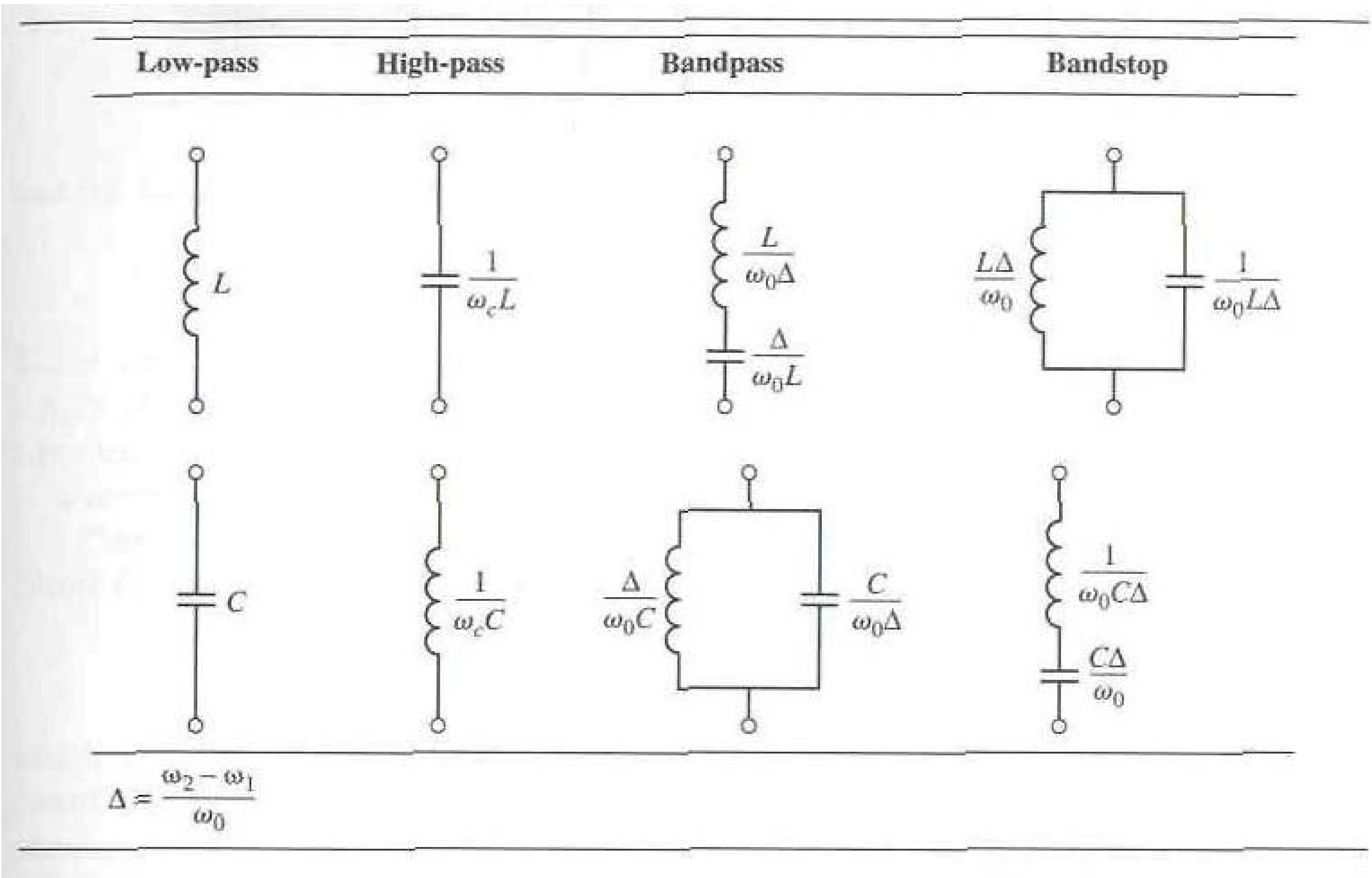
Ramura paralel

(in serie)

$$L'_k = \frac{1}{\omega_0 \Delta C_k}$$

$$C'_k = \frac{\Delta C_k}{\omega_0}$$

Transformari ale filtrului prototip

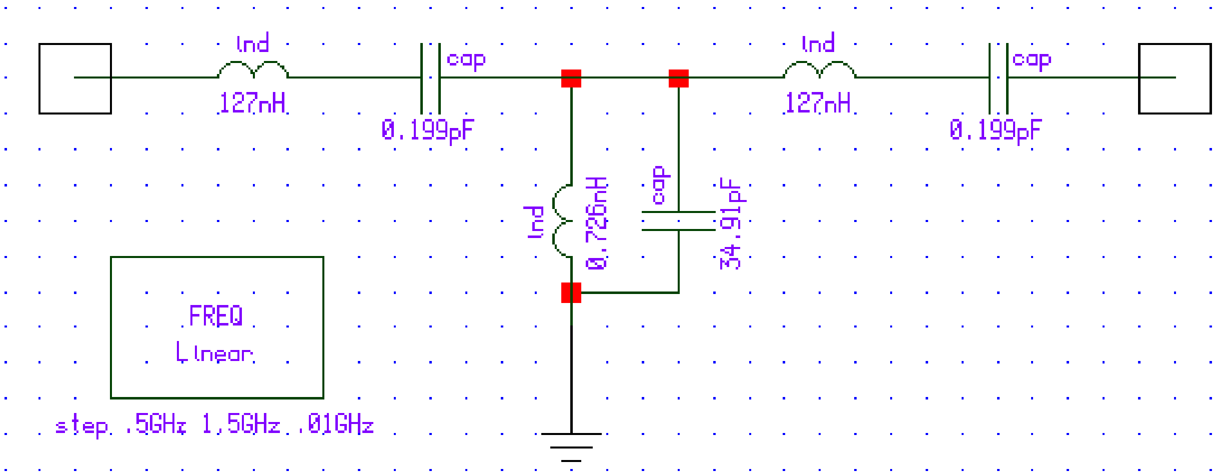


Exemplu

Să se proiecteze un filtru trece-bandă de ordinul 3, avînd riplurile în bandă de 0.5 dB. Frecvența centrala a filtrului sa fie de 1 GHz. Banda să fie de 10%, şi impedanța de 50 Ω.

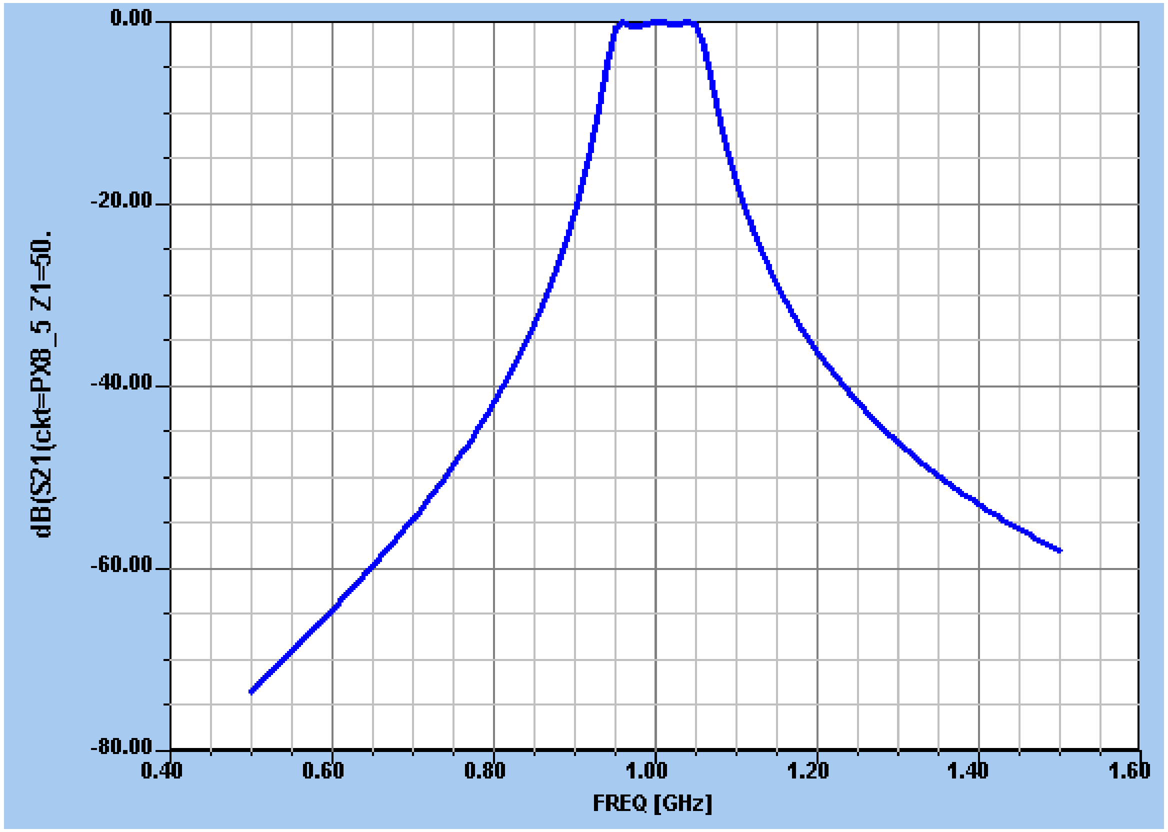
Solutie

g1 = 1.5963 = L1,
g2 **=** 1.0967 = C2,
g3 = 1.5963 = L3,
g4=1.000 =RL



L'1 = 127.0 nH,
C'l = 0.199 pF,
L'2 = 0.726 nH,
C'2 = 34.91 pF,
L'3 = 127.0 nH,
C'3 = 0.199 pF.

Simulare



dB(S21(ckt=PX8_5 Z1=50.
0.00
-20.00
-40.00
-60.00
-80.00
0.40
0.60
0.80
1.00
1.20
1.40
1.60
FREQ [GHz]

Implementarea filtrelor în domeniul microundelor

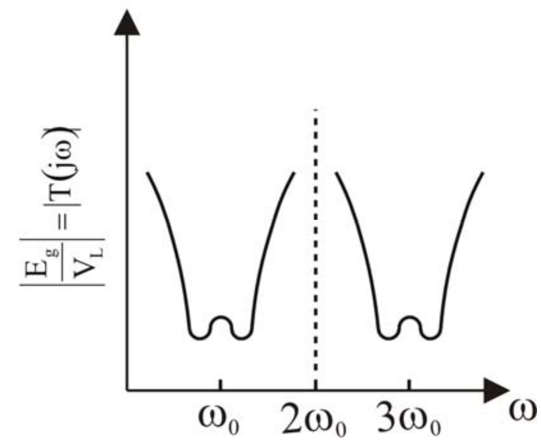
Transformarea Richard

$$\Omega = \tan(\beta l) = \tan\left(\frac{\omega l}{v_p}\right)$$

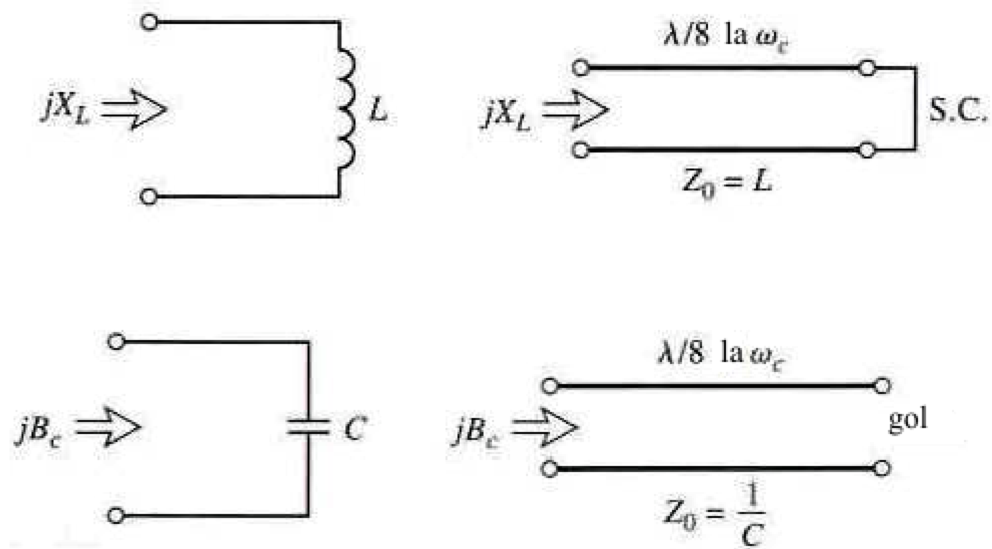
$$jX_L = j\Omega L = jL\tan(\beta l)$$

$$jB_C = j\Omega C = jC\tan(\beta l)$$

$$\Omega = 1 = \tan(\beta l)$$



Transformarea Richard

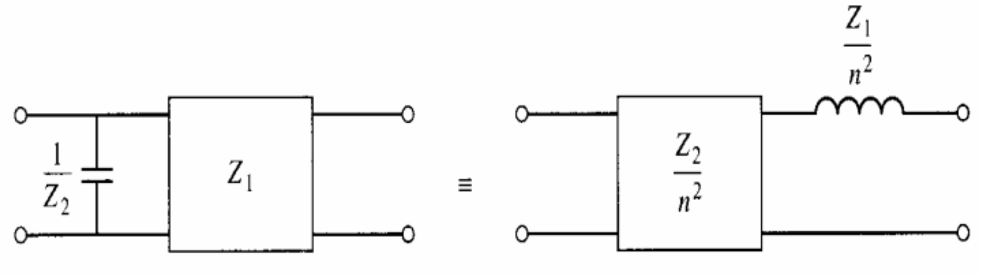


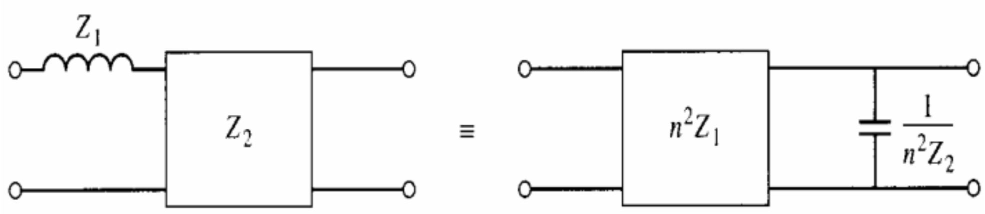
$$jX_L = j\Omega L = jL\tan(\beta l)$$

$$jB_C = j\Omega C = jC\tan(\beta l)$$

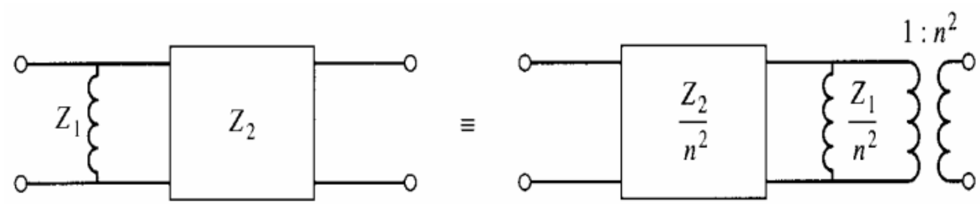
Identitățile Kuroda

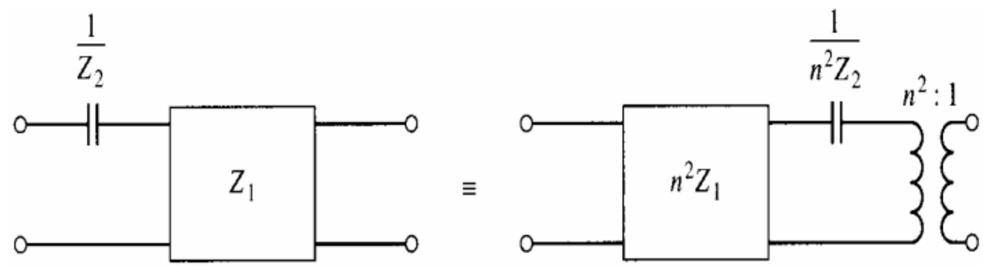
$n^2 = 1 + Z_2/Z_1$





Identitățile Kuroda $n^2 = 1 + Z_2/Z_1$





Exemplu

Să se proiecteze un filtru trece-jos în tehnologie microstrip. Specificațiile sunt: frecvența de tăiere 4 GHz, ordinul 3, impedanța de 50 Ω, şi o caracteristică echi-riplu de 3 dB.

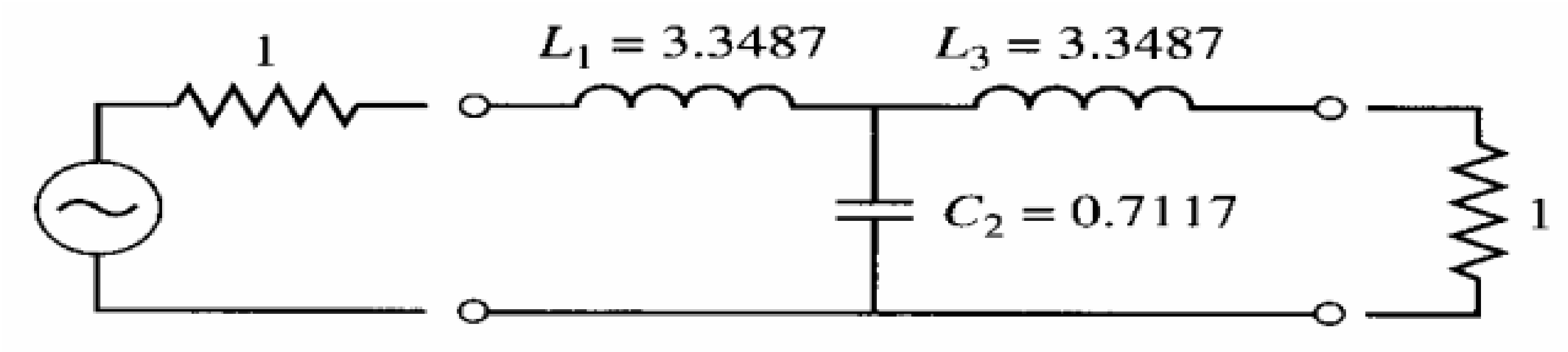
Solutie

$$g_1 = 3.3487 = L_1$$

$$g_2 = 0.7117 = C_2$$

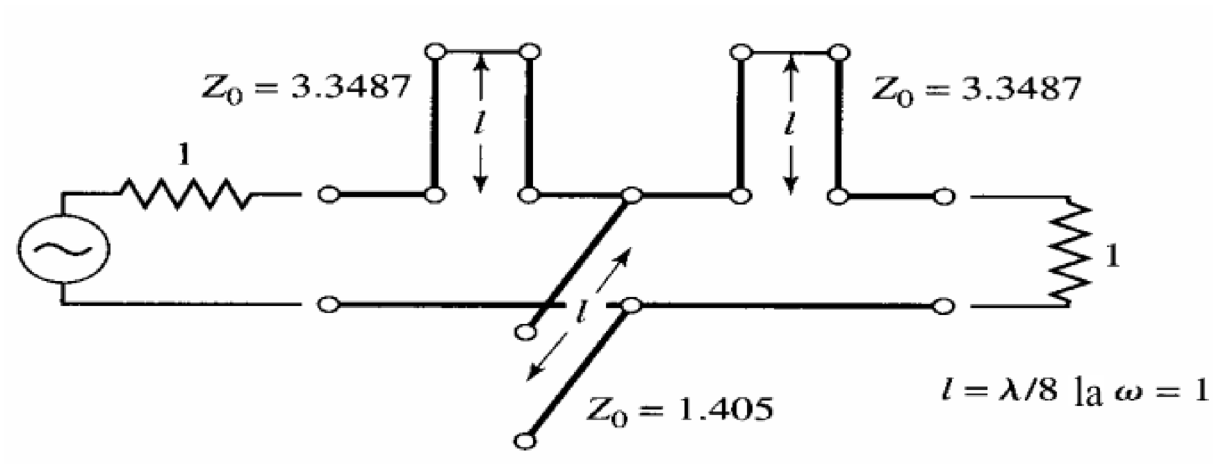
$$g_3 = 3.3487 = L_3$$

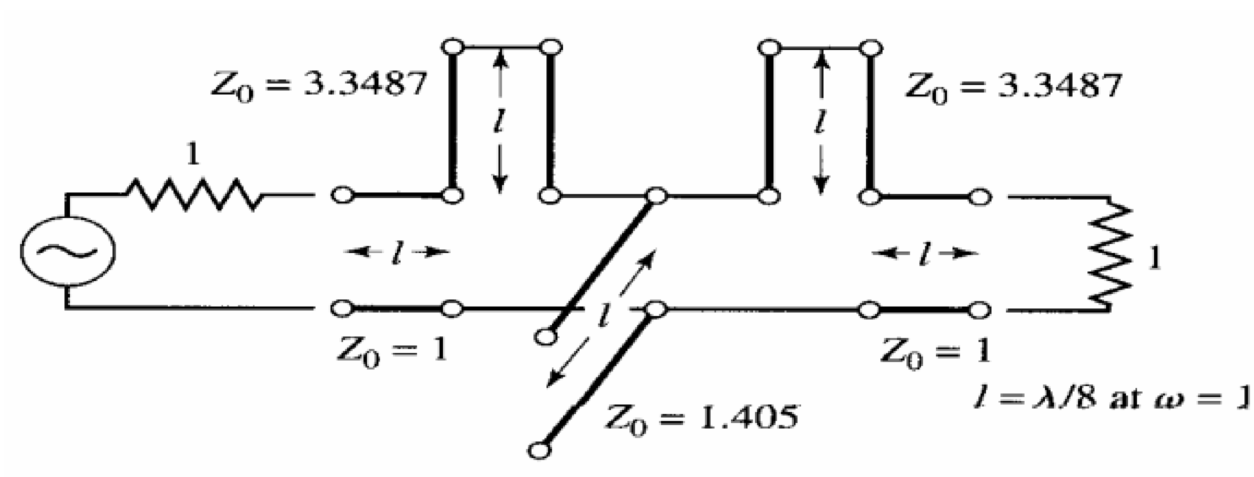
$$g_4 = 1.0000 = R_L$$



Solutie - 2

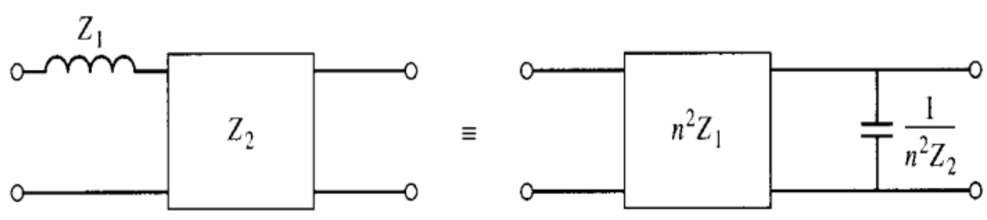
$Z_{L1} = L_1$ $\quad Z_{C2} = \frac{1}{C_2}$ $\quad Z_{L3} = L_3$

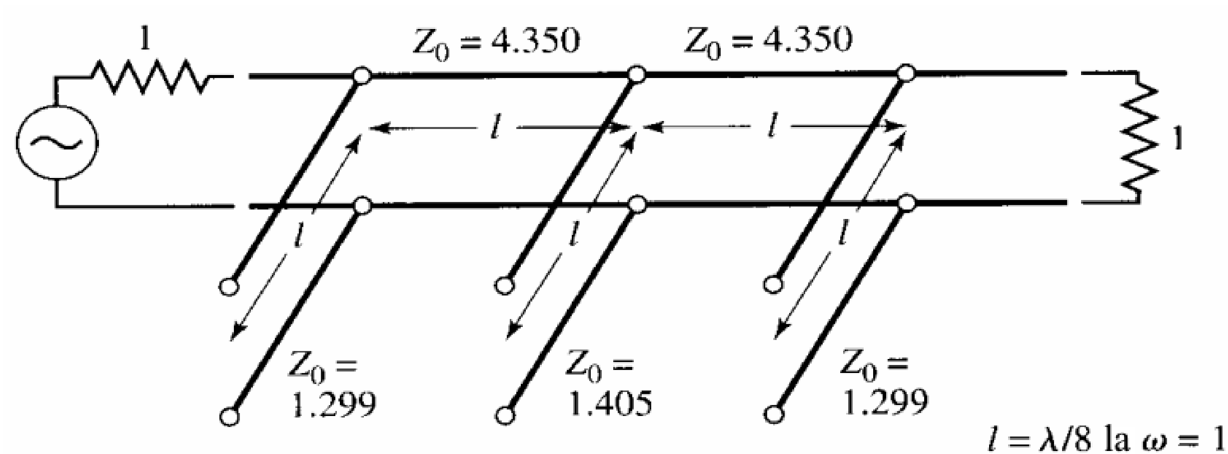




Solutie - 3

$$n = \sqrt{1 + \frac{R_L}{Z_{L1}}} \qquad n^2 = 1.299$$

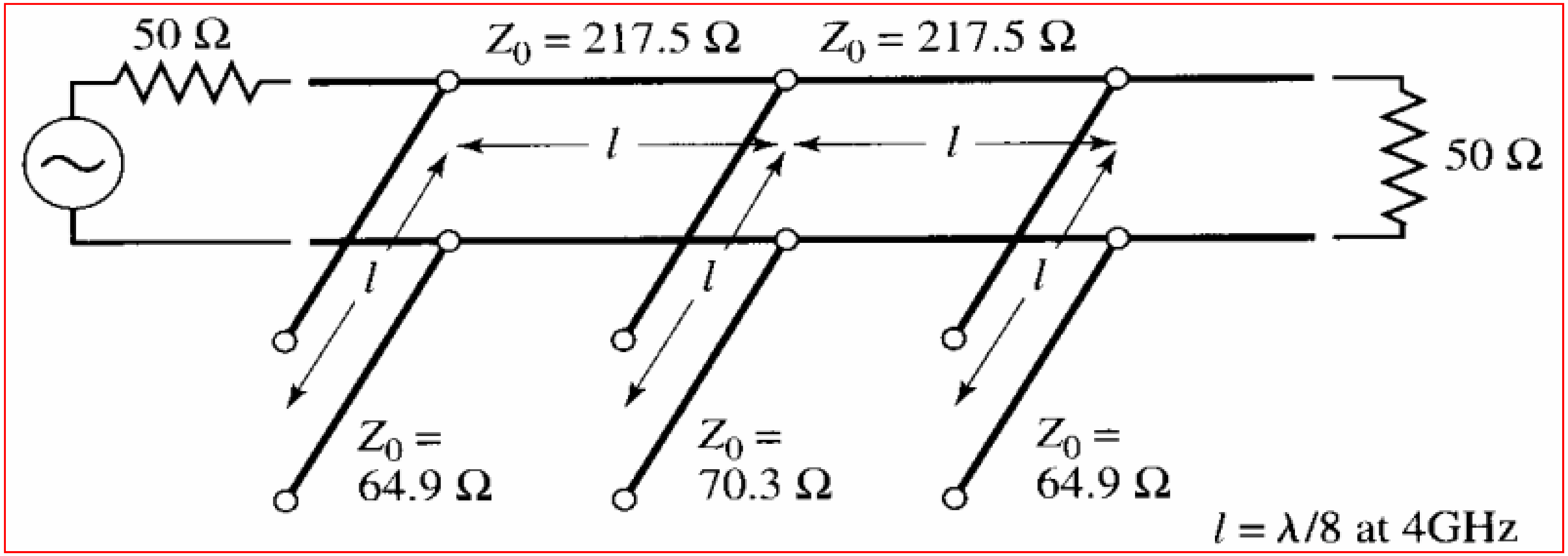




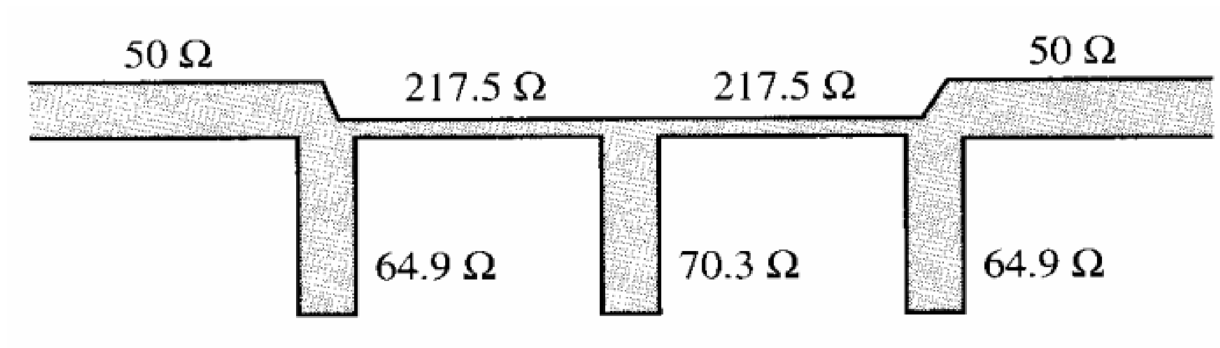
Solutie - 4

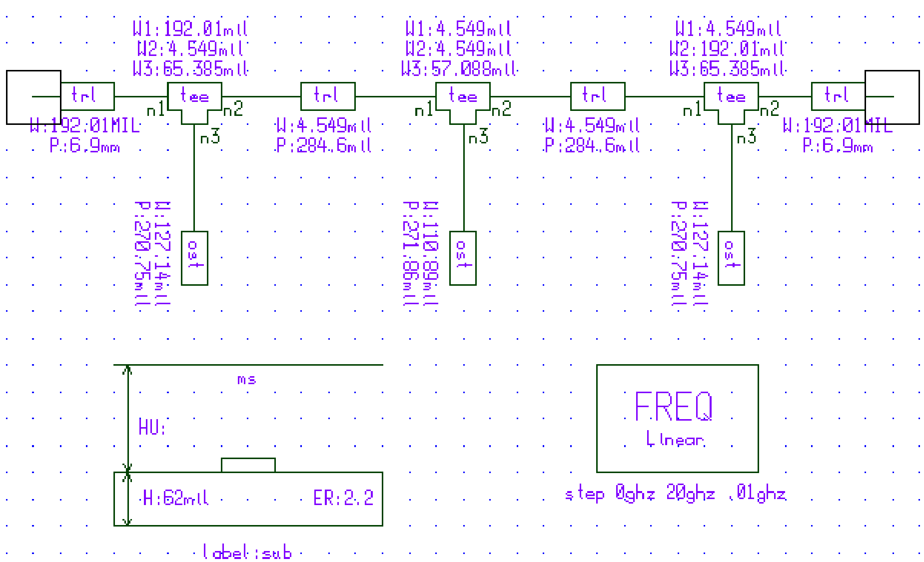
$$Z_{sh1} = n^2 Z_0 = 64.93\Omega \quad Z_{sh2} = Z_{C2} \cdot Z_0 = 70.254\Omega \quad Z_{sh3} = n^2 Z_0 = 64.93\Omega$$

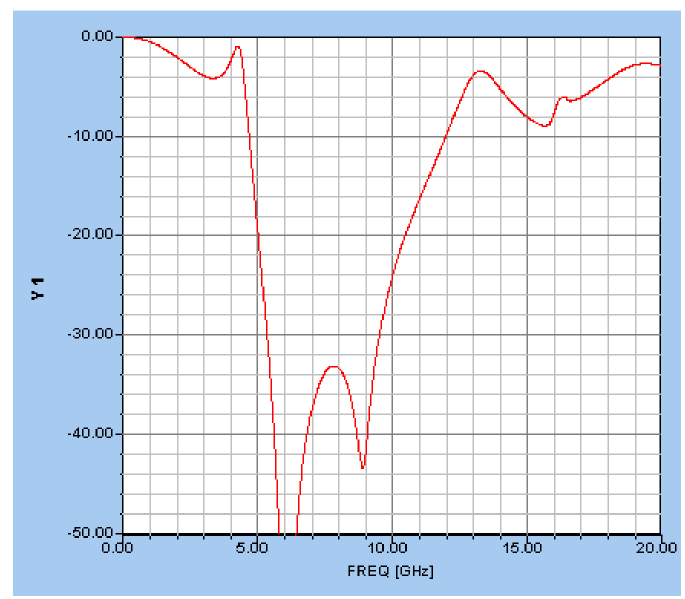
$$Z_{se1} = n^2 \cdot Z_{L1} \cdot Z_0 = 217.435\Omega \quad Z_{se2} = n^2 \cdot Z_{L3} \cdot Z_0 = 217.435\Omega$$



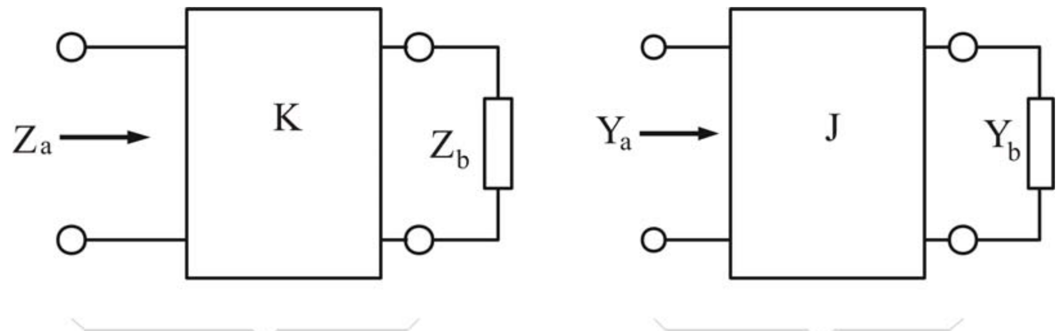
Solutie – Filtrul realizat microstrip si simulat







Invertoare de admitanță şi impedanță



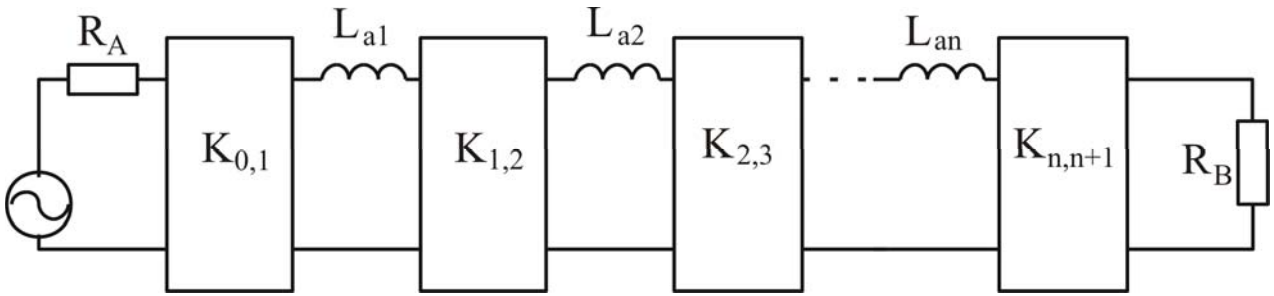
Invertor de impedanta
(a)

Invertor de admitanta
(b)

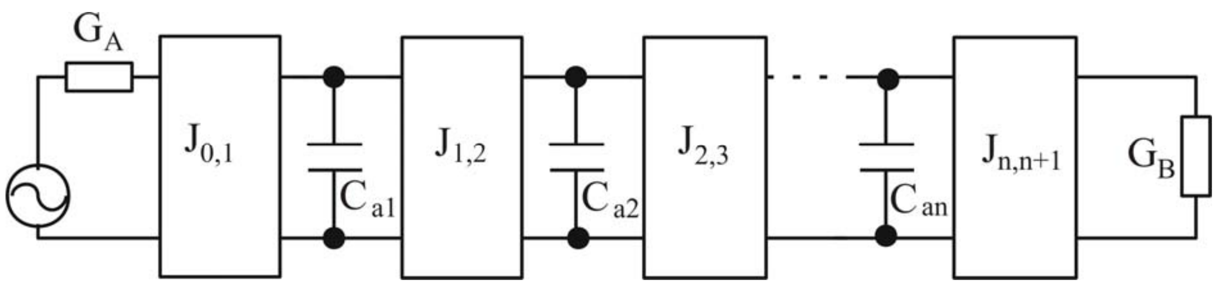
$$Z_a = \frac{K^2}{Z_b}$$

$$Y_a = \frac{J^2}{Y_b}$$

Circuitele prototip modificate folosind invertoare

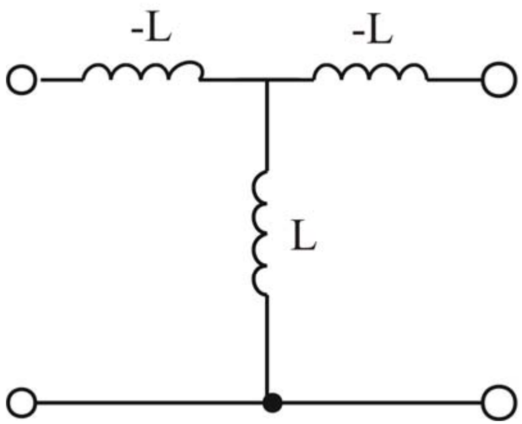


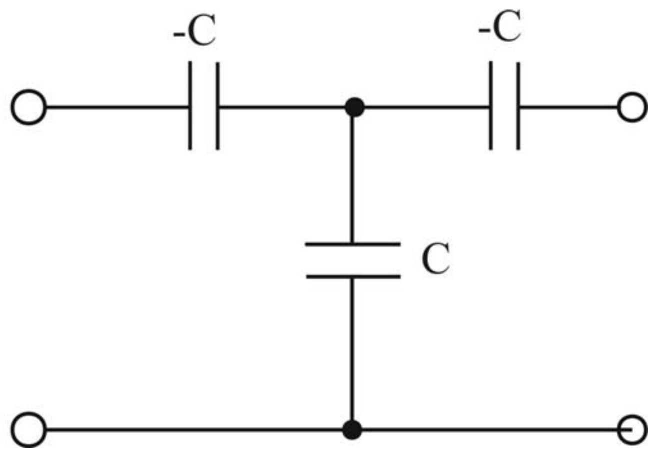
$$K_{01}=\sqrt{\frac{R_A L_{a1}}{g_0 g_1}},K_{k,k+1}\Big|_{k=\overline{1,(n-1)}}=\sqrt{\frac{L_{ak}L_{a(k+1)}}{g_k g_{k+1}}},K_{n,n+1}=\sqrt{\frac{L_{an}R_B}{g_n g_{n+1}}}$$



$$J_{01}=\sqrt{\frac{G_A C_{a1}}{g_0 g_1}},J_{k,k+1}\Big|_{k=\overline{1,(n-1)}}=\sqrt{\frac{C_{ak}C_{a(k+1)}}{g_k g_{k+1}}},J_{n,n+1}=\sqrt{\frac{C_{an}g_B}{g_n g_{n+1}}}$$

Realizări practice ale invertoarelor de impedanta- 1

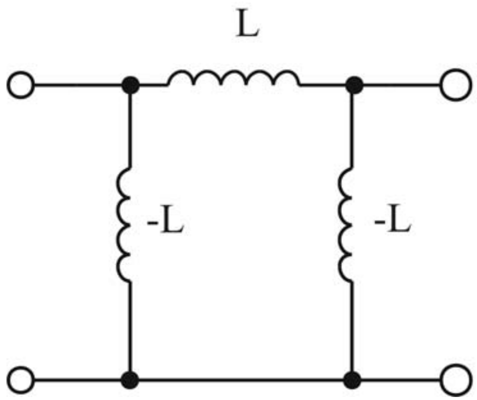


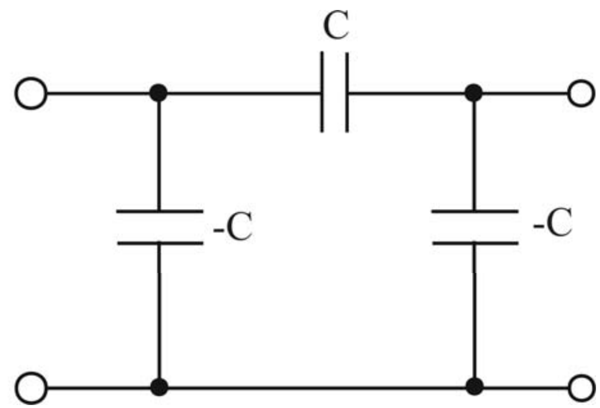


$$K = \omega L$$

$$K = 1/\omega C$$

Realizări practice ale invertoarelor de admitanta - 1

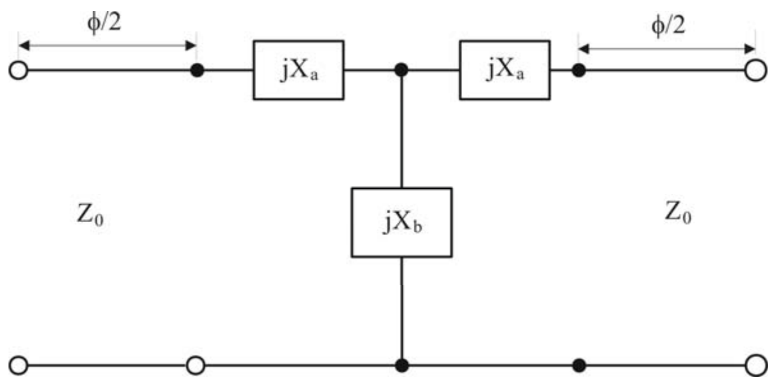




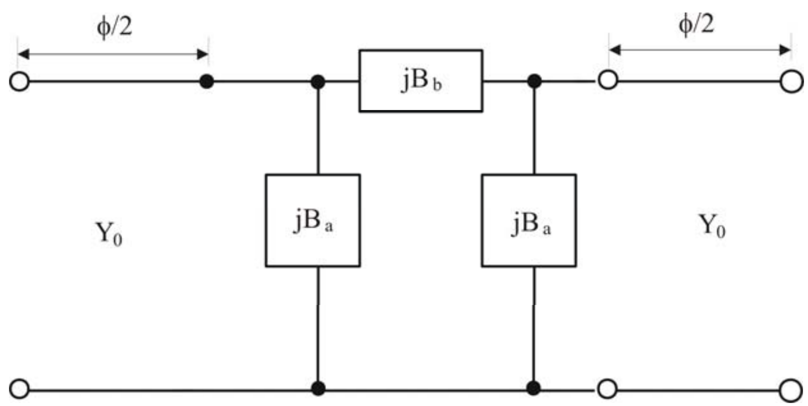
$$J = 1/\omega L$$

$$J = \omega C$$

Realizări practice ale invertoarelor de imitanta- 2

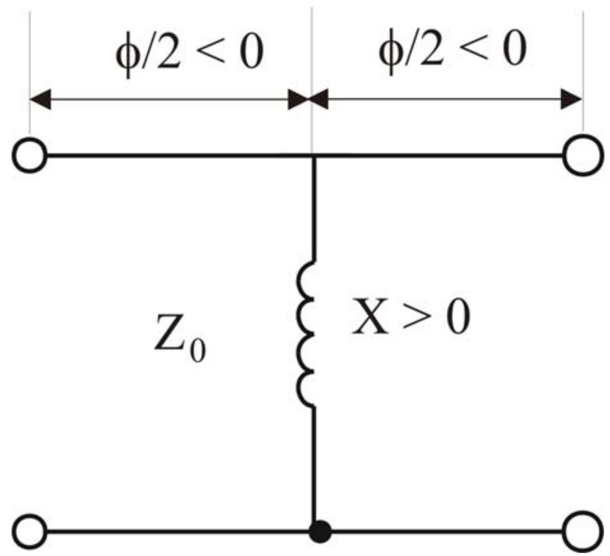


$$K = Z_0 \left| tg\left(\frac{\phi}{2} + arctg \frac{X_a}{Z_0} \right) \right| (\Omega), \phi = -arctg\left(\frac{2X_b}{Z_0} + \frac{X_a}{Z_0} \right) - arctg \frac{X_a}{Z_0} \; (rad)$$

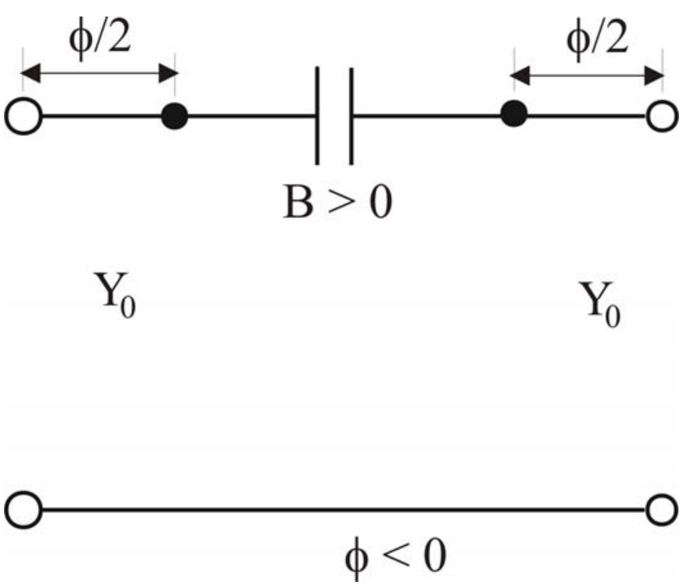


$$J = Y_0 \left| tg\left(\frac{\phi}{2} + arctg \frac{B_a}{Y_0} \right) \right| (S), \phi = -arctg\left(\frac{2B_b}{Y_0} + \frac{B_a}{Y_0} \right) - arctg \frac{B_a}{Y_0} \; (rad)$$

Realizări practice ale invertoarelor de imitanță - 3



$$K = Z_0 \mathrm{tg}\left|\frac{\phi}{2}\right| (\Omega), \quad \phi = -\mathrm{arctg}\left(\frac{2X}{Z_0}\right) (\mathrm{rad}), \quad \left|\frac{X}{Z_0}\right| = \frac{K/Z_0}{1-(K/Z_0)^2}$$

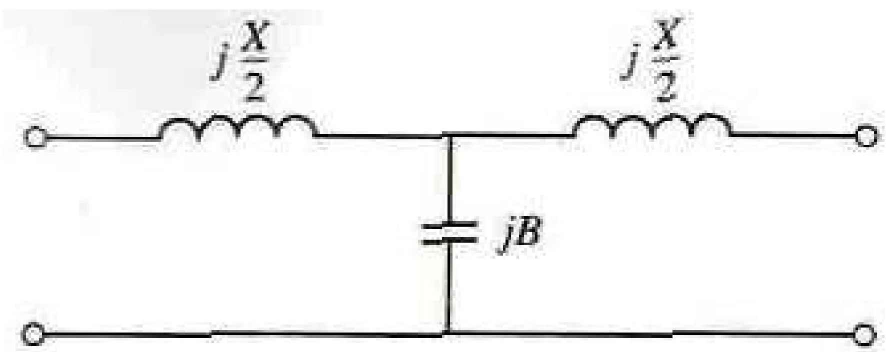


$$J = Y_0 \mathrm{tg}\left|\frac{\phi}{2}\right| (S), \quad \phi = -\mathrm{arctg}\left(\frac{2B}{Y_0}\right) (\mathrm{rad}), \quad \left|\frac{B}{Y_0}\right| = \frac{J/Y_0}{1-(J/Y_0)^2}$$

Circuit echivalent pentru sectiuni scurte de linii

$$[Z]=\begin{bmatrix} -jZ_0\cot(\beta l) & -jZ_0\cos c(\beta l) \\ -jZ_0\cos c(\beta l) & -jZ_0\cot(\beta l) \end{bmatrix}$$

$$Z_{11}-Z_{12}=-jZ_0\left[\frac{\cos(\beta l)-1}{\sin(\beta l)}\right]=jZ_0\tan\left(\frac{\beta l}{2}\right)$$



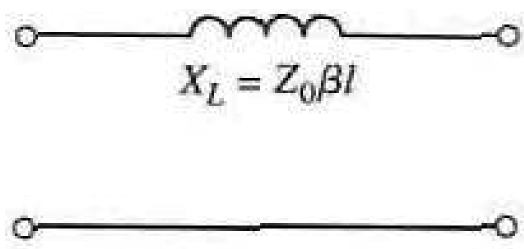
$$\beta l<\pi/2$$

$$\frac{X}{2}=Z_0\tan\left(\frac{\beta l}{2}\right)$$

$$B=\frac{1}{Z_0}\sin(\beta l)$$

Filtre trece-jos cu variații treaptă ale impedanței caracteristice

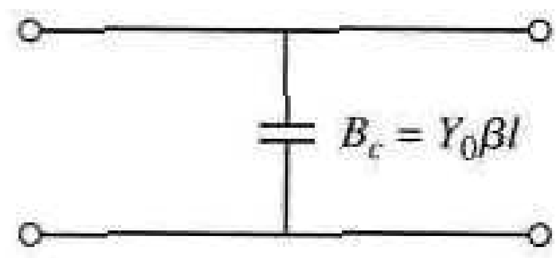
Circuite aproximativ echivalente pentru sectiuni scurte de linie



$$X \cong Z_0 \beta l$$

$$\beta l < \pi/4$$

$$Z_0 = Z_h$$



$$B \cong Y_0 \beta l$$

$$\beta l < \pi/4$$

$$Z_0 = Z_l$$

$$\beta l = \frac{L R_0}{Z_h}$$ (bobină)

$$\beta l = \frac{C Z_l}{R_0}$$ (condensator)

Exemplu

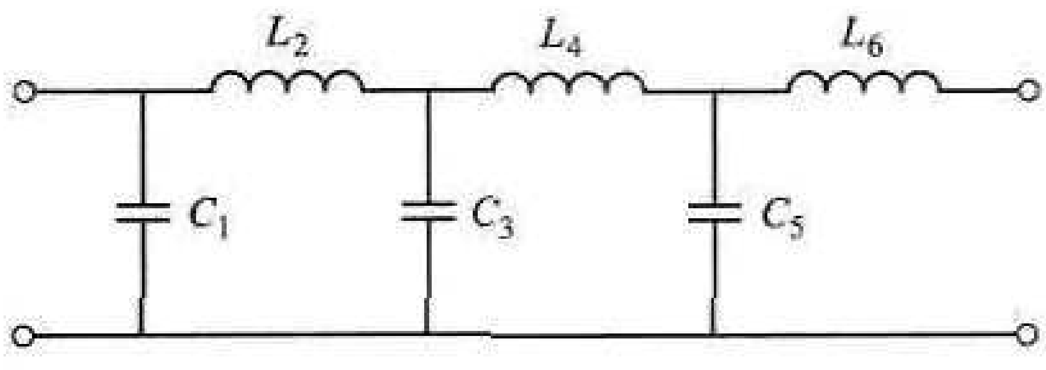
Să se proiecteze un filtru trece-jos avînd un răspuns maxim-plat, frecvența de tăiere 2.5 GHz. Este necesar să avem mai mult de 20 dB peirderi de inserție la 4 GHz. Impedanța filtrului este 50Ω, cea mai mare impedanță caracteristică realizabilă practic este 150Ω, iar cea mai mică 10Ω.

Solutia - 1

$$L_{As} = 20dB \qquad L_{Ar} = 3dB \qquad \omega'_S/\omega'_1 = 4.0/2.5 = 1.6$$

$$N = 6$$

$$g_1 = 0.517 = C_1$$
$$g_2 = 1.414 = L_2$$
$$g_3 = 1.932 = C_3$$
$$g_4 = 1.932 = L_4$$
$$g_5 = 1.414 = C_5$$
$$g_6 = 0.517 = L_6$$



Solutia - 2

$$\beta l_1 = g_1 \frac{Z_l}{R_0} = 5.9°$$

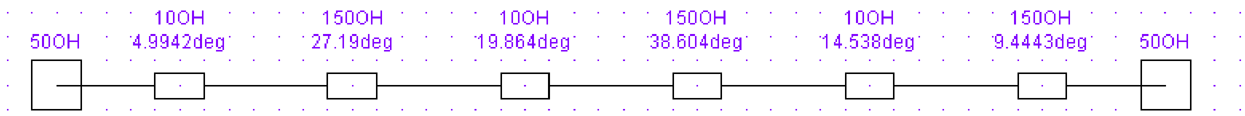
$$\beta l_2 = g_2 \frac{R_0}{Z_h} = 27.0°$$

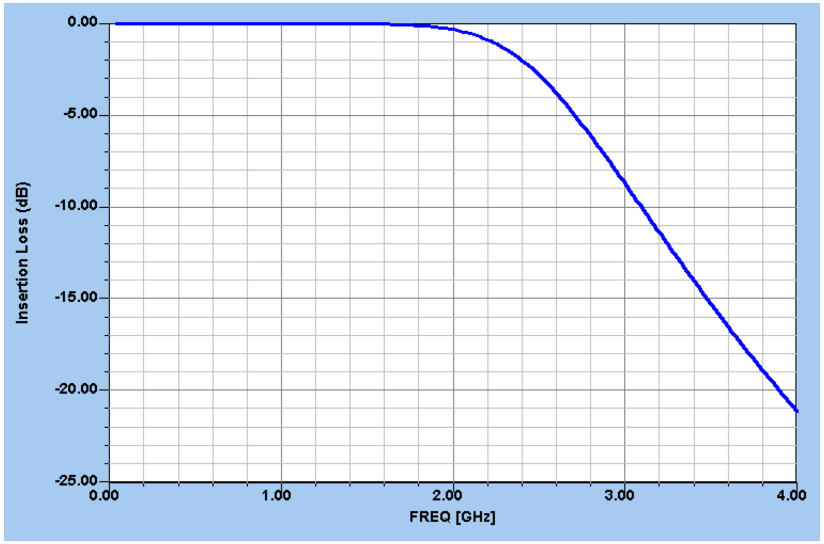
$$\beta l_3 = g_3 \frac{Z_l}{R_0} = 22.1°$$

$$\beta l_4 = g_4 \frac{R_0}{Z_h} = 36.9°$$

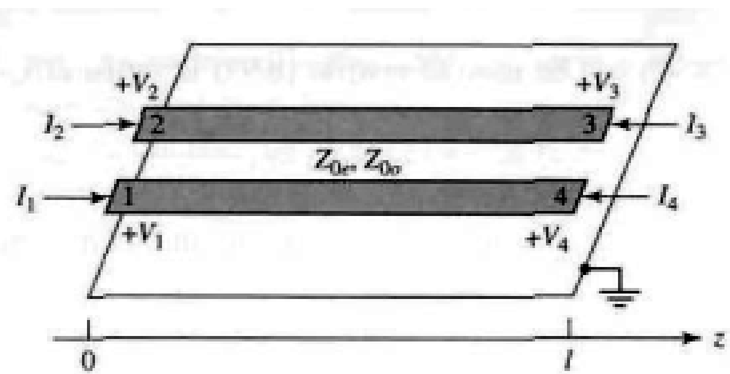
$$\beta l_5 = g_5 \frac{Z_l}{R_0} = 16.2°$$

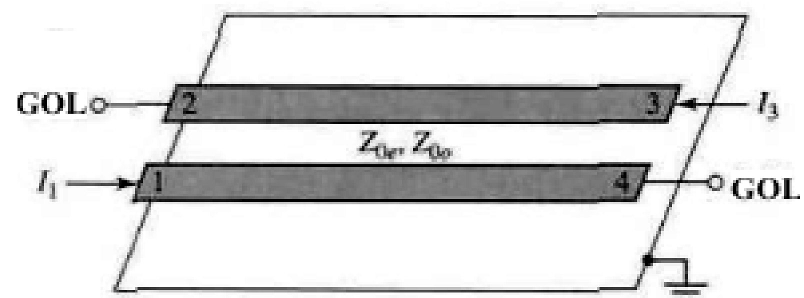
$$\beta l_6 = g_6 \frac{R_0}{Z_h} = 9.9°$$





Proiectarea filtrelor cu linii cuplate



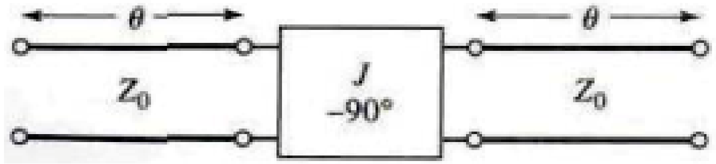


$$V_1 = Z_{11}I_1 + Z_{13}I_3$$

$$V_3 = Z_{31}I_1 + Z_{33}I_3$$

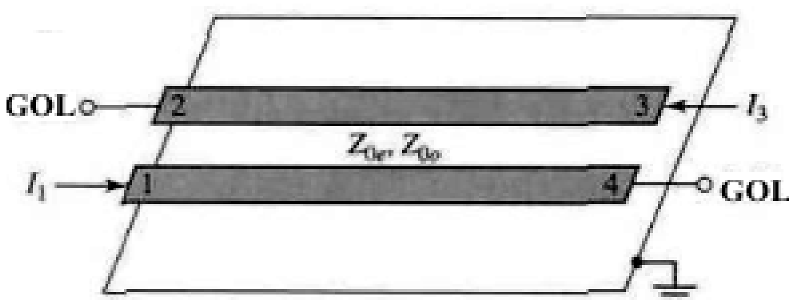
$$Z = \begin{bmatrix} \frac{-j}{2}(Z_{0e}+Z_{0o})\cot\theta & \frac{-j}{2}(Z_{0e}-Z_{0o})\cot\theta & \frac{-j}{2}(Z_{0e}-Z_{0o})\csc\theta & \frac{-j}{2}(Z_{0e}+Z_{0o})\csc\theta \\ \frac{-j}{2}(Z_{0e}-Z_{0o})\cot\theta & \frac{-j}{2}(Z_{0e}+Z_{0o})\cot\theta & \frac{-j}{2}(Z_{0e}+Z_{0o})\csc\theta & \frac{-j}{2}(Z_{0e}-Z_{0o})\csc\theta \\ \frac{-j}{2}(Z_{0e}-Z_{0o})\csc\theta & \frac{-j}{2}(Z_{0e}+Z_{0o})\csc\theta & \frac{-j}{2}(Z_{0e}+Z_{0o})\cot\theta & \frac{-j}{2}(Z_{0e}-Z_{0o})\cot\theta \\ \frac{-j}{2}(Z_{0e}+Z_{0o})\csc\theta & \frac{-j}{2}(Z_{0e}-Z_{0o})\csc\theta & \frac{-j}{2}(Z_{0e}-Z_{0o})\cot\theta & \frac{-j}{2}(Z_{0e}+Z_{0o})\cot\theta \end{bmatrix}$$

Proiectarea unui filtru trece-bandă cu linii cuplate



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_0\sin\theta \\ \dfrac{j\sin\theta}{Z_0} & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} 0 & -j/J \\ -jJ & 0 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & jZ_0\sin\theta \\ \dfrac{j\sin\theta}{Z_0} & \cos\theta \end{bmatrix} =$$

$$\begin{bmatrix} \left(JZ_0 + \dfrac{1}{JZ_0}\right)\sin\theta\cos\theta & j\left(JZ_0^2\sin^2\theta - \dfrac{\cos^2\theta}{J}\right) \\ j\left(\dfrac{1}{JZ_0^2}\sin^2\theta - J\cos^2\theta\right) & \left(JZ_0 + \dfrac{1}{JZ_0}\right)\sin\theta\cos\theta \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \dfrac{Z_{11}}{Z_{21}} & \dfrac{|Z|}{Z_{21}} \\ \dfrac{1}{Z_{21}} & \dfrac{Z_{22}}{Z_{21}} \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \dfrac{Z_{0e}+Z_{0o}}{Z_{0e}-Z_{0o}}\cos\theta & \dfrac{-j}{2}\left(\dfrac{(Z_{0e}+Z_{0o})^2}{Z_{0e}-Z_{0o}}\cos^2\theta + (Z_{0e}-Z_{0o})\dfrac{1}{\sin\theta}\right) \\ 2j\dfrac{1}{Z_{0e}-Z_{0o}}\sin\theta & \dfrac{Z_{0e}+Z_{0o}}{Z_{0e}-Z_{0o}}\cos\theta \end{bmatrix}$$

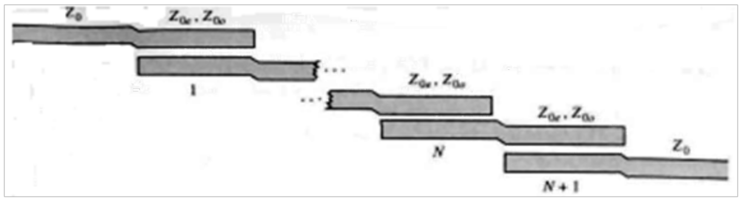
$$\frac{1}{2}(Z_{0e}-Z_{0o}) = JZ_0^2$$

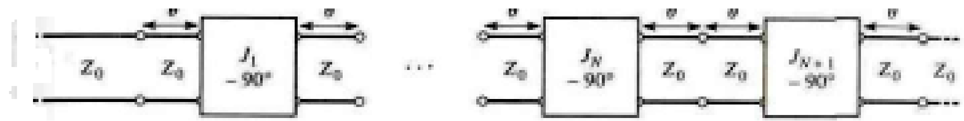
$$\frac{Z_{0e}+Z_{0o}}{Z_{0e}-Z_{0o}} = JZ_0 + \frac{1}{JZ_0}$$

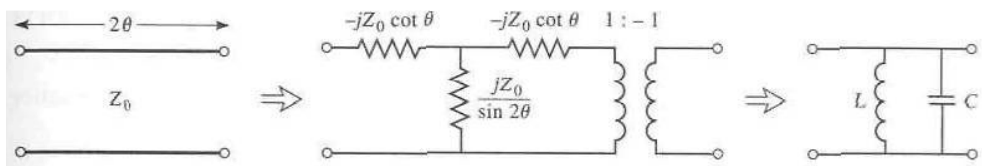
$$Z_{0e} = Z_0\left[1 + JZ_0 + (JZ_0)^2\right]$$

$$Z_{0o} = Z_0\left[1 - JZ_0 + (JZ_0)^2\right]$$

Calculul sectiunilor interne







$$2\theta = \beta l = \omega l / v_p = (\omega_0 + \Delta\omega)\pi/\omega_0 = \pi(1 + \Delta\omega/\omega_0)$$

$$\theta \approx \pi/2 \qquad Z_{12} = \frac{jZ_0}{\sin \pi(1+\Delta\omega/\omega_0)} \approx \frac{-jZ_0\omega_0}{\pi(\omega-\omega_0)}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{Z_{11}}{Z_{12}} & \frac{Z_{11}^2 - Z_{12}^2}{Z_{12}} \\ \frac{1}{Z_{12}} & \frac{Z_{11}}{Z_{12}} \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} \frac{-Z_{11}}{Z_{12}} & \frac{Z_{12}^2 - Z_{11}^2}{Z_{12}} \\ \frac{-1}{Z_{12}} & \frac{-Z_{11}}{Z_{12}} \end{bmatrix}$$

$$Z = \frac{-jL\omega_0^2}{2(\omega-\omega_0)}$$

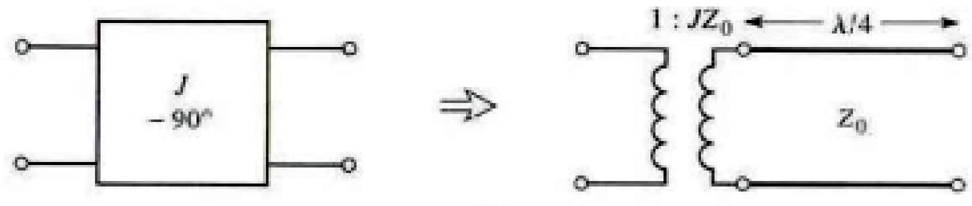
$$Z_{12} = \frac{-1}{C} = \frac{jZ_0}{\sin 2\theta}$$

$$L = \frac{2Z_0}{\pi\omega_0} \qquad C = \frac{1}{\omega_0^2 L} = \frac{\pi}{2Z_0\omega_0}$$

$$Z_{11} = Z_{22} = -Z_{12}A = -jZ_0 \cot 2\theta$$

$$Z_{11} - Z_{12} = -jZ_0 \frac{1+\cos 2\theta}{\sin 2\theta} = -jZ_0 \cot\theta$$

Calculul sectiunilor de capat



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & -j/J \\ -jJ & 0 \end{bmatrix}$$

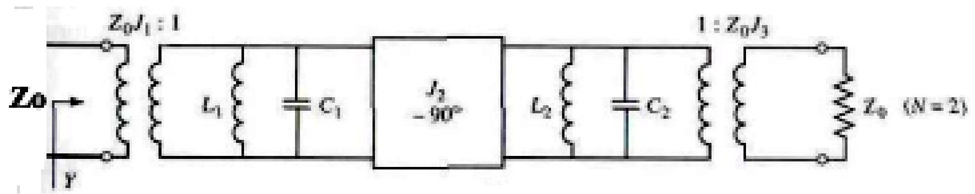
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{1}{N} & 0 \\ 0 & N \end{bmatrix} \cdot \begin{bmatrix} 0 & -jZ_0 \\ \frac{-j}{Z_0} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-jZ_0}{N} \\ \frac{-jN}{Z_0} & 0 \end{bmatrix}$$

$$N = JZ_0$$

Circuitul echivalent al filtrului

$$Y=\frac{1}{J_1^2Z_0^2}\left\{j\omega C_1+\frac{1}{j\omega L_1}+\frac{J_2^2}{j\sqrt{C_2/L_2}\left[(\omega/\omega_0)-(\omega_0/\omega)\right]+Z_0J_3^2}\right\}=$$

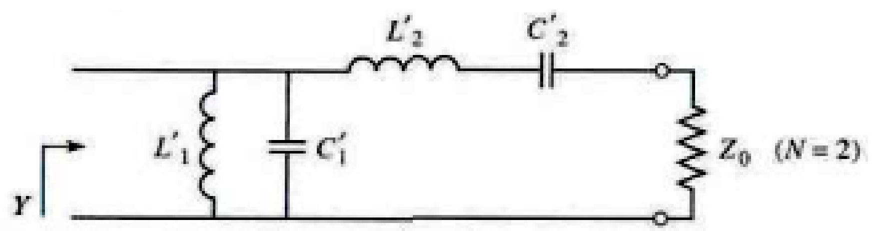
$$=\frac{1}{J_1^2Z_0^2}\left\{j\sqrt{\frac{C_1}{L_1}}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)+\frac{J_2^2}{j\sqrt{C_2/L_2}\left[(\omega/\omega_0)-(\omega_0/\omega)\right]+Z_0J_3^2}\right\}$$



$$j\omega C_2+\frac{1}{j\omega L_2}+Z_0J_3^2=j\sqrt{\frac{C_2}{L_2}}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)+Z_0J_3^2$$

$$Y=j\omega C_1'+\frac{1}{j\omega L_1'}+\frac{1}{j\omega L_2'+(1/j\omega C_2')+Z_0}=$$

$$=j\sqrt{\frac{C_1'}{L_1'}}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)+\frac{1}{j\sqrt{L_2'/C_2'}\left[(\omega/\omega_0)-(\omega_0/\omega)\right]+Z_0}$$



$$\frac{1}{J_1^2Z_0^2}\sqrt{\frac{C_1}{L_1}}=\sqrt{\frac{C_1'}{L_1'}}\qquad\frac{J_1^2Z_0^2}{J_2^2}\sqrt{\frac{C_2}{L_2}}=\sqrt{\frac{L_2'}{C_2'}}\qquad\frac{J_1^2Z_0^3J_3^2}{J_2^2}=Z_0$$

Relatiile de calcul ale filtrului

$$L'_1 = \frac{\Delta Z_0}{\omega_0 g_1}$$

$$C'_1 = \frac{g_1}{\Delta \omega_0 Z_0}$$

$$L'_2 = \frac{g_2 Z_0}{\Delta \omega_0}$$

$$C'_2 = \frac{\Delta}{\omega_0 g_2 Z_0}$$

$$L_n = \frac{2Z_0}{\pi \omega_0}$$

$$C_n = \frac{1}{\omega_0^2 L_n} = \frac{\pi}{2 Z_0 \omega_0}$$

$$J_1 Z_0 = \left(\frac{C_1 L'_1}{L_1 C'_1}\right)^{1/4} = \sqrt{\frac{\pi\Delta}{2g_1}}$$

$$J_2 Z_0 = J_1 Z_0^2 \left(\frac{C_2 C'_2}{L_2 L'_2}\right)^{1/4} = \frac{\pi\Delta}{2\sqrt{g_1 g_2}}$$

$$J_3 Z_0 = \frac{J_2}{J_1} = \sqrt{\frac{\pi\Delta}{2g_2}}$$

$$\Delta = (\omega_2 - \omega_1)/\omega_0$$

$$Z_0 J_1 = \sqrt{\frac{\pi\Delta}{2g_1}}$$

$$Z_0 J_n = \frac{\pi\Delta}{2\sqrt{g_{n-1} g_n}} \qquad n = 2,3,\ldots,N$$

$$Z_0 J_{N+1} = \sqrt{\frac{\pi\Delta}{2 g_N g_{N+1}}}$$

$$(Z_{0e})_k = Z_0 \left[1 + J_k Z_0 + (J_k Z_0)^2\right]$$

$$(Z_{0o})_k = Z_0 \left[1 - J_k Z_0 + (J_k Z_0)^2\right]$$

Exemplu

Proiectaţi un filtru trece-bandă cu N=3 şi ripluri de 0.5 dB în bandă. Frecvenţa centrală este de 2 GHz, banda de 10% şi

$$Z_0 = 50\Omega$$

. Care este atenuarea la 1.8 GHz ?

Solutie

$$\omega \leftarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)=\frac{1}{0.1}\left(\frac{1.8}{2.0}-\frac{2.0}{1.8}\right)=-2.11$$

$$L_A(\text{dB})=10\log\left[1+\varepsilon_r\left(\text{ch}^2 n\left(\text{arcch}\left(\frac{\omega_s'}{\omega_1'}\right)\right)\right)\right]=$$

$$10\log\left[1+0.122\left(\text{ch}^2 3\left(\text{arcch}\left(2.11\right)\right)\right)\right]=20.8\text{ dB}$$

n	g_n	$Z_0 J_n$	$Z_{0e}(\Omega)$	$Z_{0o}(\Omega)$
1	1.5963	0.3137	70.61	39.24
2	1.0967	0.1187	56.64	44.77
3	1.5963	0.1187	56.64	44.77
4	1.0000	0.3137	70.61	39.24

Rezultatul simularii